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ABSTRACT

Mathematics teachers need to study diverse psychologies of learning so that individual learners may be guided to attain as optimally as possible. This paper discusses meaning theory in mathematics teaching and describes a variety of learning theories such as behaviorism, developmental psychology, and problem solving. (ASK)

PSYCHOLOGICAL FOUNDATIONS IN TEACHING MATHEMATICS

Mathematics teachers need to study diverse psychologies of learning for use so that individual learners may be guided to attain as optimally as possible. With a thorough knowledge of the psychology of learning, teachers may do a better job of teaching mathematics to pupils of all ability levels. Individual differences among learners must be provided for in order that each pupil may learn as much mathematics as possible. A quality mathematics teacher emphasizes objectives, learning opportunities, and appraisal procedures that assist pupils individually to perceive meaning in the mathematics curriculum.

Meaning Theory in Teaching Mathematics

Mathematics teachers need to be certain that each pupil attaches meaning to facts, concepts, and generalizations acquired in the curriculum. With meaningful subject matter, pupils may understand that which was taught. Understanding content presented in mathematics assists the pupil to clarify information. Clarity of understanding stresses that pupils comprehend subject matter presented. In deductive teaching, the teacher explains each fact, concept, and generalization so that meaningful learning may accrue. When induction as a method of instruction is used, the mathematics teacher asks numerous questions of pupils to receive answers and feedback if they understand what was taught. Teachers must spend adequate time for depth instruction in guiding pupils to perceive meaning, deductively and inductively, through the use of concrete, semiconcrete, and abstract materials of instruction in terms of content taught.

Directly related to pupils attaching meaning to ongoing content presented, leaner must be able to use subject matter acquired. If subject matter acquired is used, the chances are it will be retained better than if it were not used. The teacher should assist pupils to apply facts, concepts, and generalizations acquired. Thus information can be used in solving word problems from the basal textbook used in the classroom. Pupils may also use previously acquired content when solving life-like problems in mathematics. The teacher might write problems and photocopy them for learners to respond to. These problems are directly related to what has been taught in ongoing lessons and units in mathematics. Learners too may write problems and exchange papers with others to solve each problem so that application can be made of what has been learned previously. Discussions led by the teacher can also get pupils wholeheartedly involved in making use of subject matter learned.

The mathematics teacher needs to guide learners to engage in higher levels of cognition when using meaningful materials for learners. Thus pupils need to have opportunities to engage in critical thinking. In separating the relevant from the irrelevant in working to secure answers to a word problem in mathematics involves critical thought. A separation then of what is salient as compared to that which is not important is necessary in critical thinking. That which is significant is then used to obtain an answer to a word problem contained in the basal or written by the teacher or a learner. Life in society demands that pupils become proficient now and in the future as an adult in the area of critical thought. Separating reality form fantasy and the real from that which is imaginary are necessary ingredients in critical thinking.

Novel solutions are needed to solve numerous problems. Creative thinking then needs adequate emphasis in the mathematics curriculum. For example, to solve a word problem, several algorithms may be used to arrive at an answer. Each algorithm might well provide the correct answer. Learners should become familiar with diversity involved here so that the algorithm that works best for the pupil may be used. Unique solutions to word problems should assist pupils to explore different options to each problem in the societal arena.

Hopefully, this transfer from the mathematics curriculum to the real world of society will be in evidence. In society, individuals meet up with unique situations in which solutions are needed that are different from any solution used in the past.

Life-like problems actually faced by learners in mathematics need adequate emphasis in the curriculum. These problems involve buying and selling items, how to stay within one's own budget, as well as an increased use of mathematics in society emphasize problem solving involving reality. A creative mind may be necessary presently for the pupil as well as in the future to solve these problems involving mathematics. Creative thinking must be a definite goal in the mathematics curriculum. Critical thought is also necessary for pupils in solving life-like problems in mathematics. Comparing solutions in answer to a problem certainly stresses critical thought. Further situations involving critical thought emphasizes analyzing a problem in mathematics to study component parts. After comparing and analyzing possible solutions in dealing with problems in mathematics in the real world, a synthesis is needed. To synthesize, creative thinking again is in evidence. Synthesizing emphasizes securing wholeness in coming up with a solution to a problem area. Steps inherent in solving life-like problems (or word problems) in mathematics include the following:

1. defining the problem whereby clarity is in evidence.
2. gathering information in arriving at a tentative solution.
3. developing a hypothesis based on the acquired information.
4. testing the hypothesis in step three above.
5. revising the hypothesis, if needed.

In addition to pupils attaching meaning to what has been learned, applying that which has been learned, and engaging in higher levels of cognition, pupils also need to perceive purpose in learning in ongoing lessons and units off study in mathematics. There are selected approaches which can be used by the mathematics teacher to guide pupils to perceive purpose for achieving. A deductive procedure might be used. With deduction, the mathematics teacher explains to learners why the subject matter to be studied is relevant. I believe that the small amount of time needed to explain to learners why the subject matter to be acquired is salient is time well spent in teaching and learning situations. Instead of a deductive approach in guiding pupils to perceive purpose or reasons for learning, there are teachers who prefer an inductive approach. Here, the mathematics teacher asks questions of pupils as to why they believe the content to be studied is relevant to learn. Inductive procedures are more time consuming as compared to deduction since responses must come form learners when determining the relevance in studying vital facts, concepts, and generalizations in mathematics.

A third approach in guiding pupils to perceive purpose in learning is to use extrinsic rewards. Here, the mathematics teacher needs to announce prior to instruction what pupils are to learn as well as the reward that will accrue to learners if they achieve this goal. Rewards to be given might be inexpensive prizes, tokens to be exchanged for prizes, time given for a self selected activity, or extra recess time. The reward must motivate pupils to achieve more optimally in mathematics. It is given to pupils only if they have attained a goal announced by the teacher prior to lesson presentation. The amount of learning that must be acquired before the pupil secures the reward is determined by the teacher. The working for the reward is a motivator for the learner. Receiving the reward for goal attainment is a reinforcer to encourage similar future behavior.

Interest is a powerful factor in learning. The mathematics teacher needs to obtain the attention of all learners during teaching- learning situations. To demand attention of pupils does not capture learner interest in mathematics. Rather, the teacher needs to use a variety of materials in teaching mathematics to obtain intrinsic interests of learners. These activities include lifelike problems which need solution, textbook and workbook assignments, films, slides, video tapes, video disks, illustrations, teaching aids, technology, integrated learning systems, teaching units, as well as resource units of study in mathematics. In using a variety of learning activities, the mathematics teacher has a better chance in securing learner interest as compared to a single type of material. The tone of the teacher's voice must have appropriate voice inflection, pitch, and juncture. A monotonous tone of voice will not tend to obtain learner interest and attention. Quality eye contact with pupils should aid in obtaining pupil attention and promote learning in mathematics. Interest of pupils in ongoing lessons and units of study develops effort for achieving.

Theories of Learning in Mathematics

Selected theories of learning in educational psychology used by the teacher should assist pupils to attain at a more optimal level. Operant conditioning, as developed by B.F. Skinner (1904- 1988), has done much as a theory of learning to guide learner progress.
Dr. Skinner stressed the use of programmed learning in emphasizing behaviorism as a psychology of learning. Here, a qualified programmer would determine what pupils are to learn in any unit of study in mathematics.The body of knowledge within the unit is broken down into component parts. The steps of attainment are very small when working on a program in mathematics, be it in textbook or software form. The pupil, here, generally reads a sentence or two, depending on the maturity level of the involved learner. He/she then views a related illustration, responds to a test item, and checks the response. If correct, the pupil is rewarded. If incorrect, the pupil now knows the correct answer and is also ready for the next sequential programmed item. The program has been tried out previously in pilot studies with needed modifications made. The content to follow in each program moves from the simple to the increasingly more complex. The same procedure, or a slight modification, may follow in each step of learning such as read a sentence or more, view an illustration, respond to a test item, and check the correctness of the response as provided by the programmer. Answers given by the learner are either correct or incorrect. By being correct approximately 90 per cent of the time in responding, the pupil can make continuous progress with

increasingly complex items in programmed learning. A positive self concept could be an end result for pupils if they respond with an approximate 90 per cent correct in terms of accuracy. Tutorial programs using computers tend to stress tenets of programmed instruction. Simulation in computer use may also stress programmed learning, providing it is not too open ended in its subject matter presentation. B.F. Skinner believed strongly in answers being either right or wrong when learners make responses. Shankaranarayana wrote:

> For Skinner (1969), " teaching is an arrangement of contingencies of reinforcement which expedite learning." Skinner believes that promotion of learning is possible by giving attention to the following factors: the behavior that is to be learned, the reinforcers that may be used , and the scheduling of reinforcers.
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> Skinner recommends the use of programmed instruction which provides for individual differences by allowing students to achieve at their own rate of speed. In terms of Skinner' operant behaviorism, "a program can be seen as an arrangement of material that will lead students to emit correct responses and will also provide reinforcement for that response... The essential elements of programmed instruction... are (1) an ordered sequence of stimuli, (2) specific student response, (3) immediate knowledge of results, (4) small steps, (5) minimum errors, (6) gradual shaping of terminal behavior and (7) self pacing.

B. F. Skinner has a well known and popular name in education. His experiments in teaching and education have indeed been numerous. Morris and Pai (1976) wrote the following:

> As Skinner has pointed out several times, the most important task of the teacher is to arrange conditions under which desired learning can occur. Considering the fact that teachers are to bring about changes in extremely complex behavior, they should be specialists in human behavior. Effective and efficient manipulation of the multitude of variables affecting children's intellectual and social behaviors cannot be accomplished by trial and error alone, nor should such work be based solely on the personal experiences of the teacher, since this covers only a limited range of circumstances.. Consequently, a scientific study of human behavior is vital in the improvement of teaching, because it provides us with accurate and reliable knowledge about learning and leads to the development of new instructional materials, methods, and techniques. Similarly, an empirical analysis of the teaching process is essential, for it clarifies the teacher's

responsibility through a series of small and progressive approximations. This approach makes teaching practices more specific, thereby facilitating a more effective evaluation.

James Popham and Behaviorism

James Popham from the University of California is a strong advocate of behaviorism. Dr. Popham (1970, see bibliography entries) developed a series of filmstrips and related cassette tapes proposing behaviorism as a needed central theme of teaching and learning. Behaviorists believe strongly in the use of measurably stated objectives in teaching pupils. These precise objectives are written prior to teaching learners. Ideally, there is no leeway in determining what will be taught when viewing the written statement of objectives. The teacher then is certain as to what will be taught. He/she may announce to pupils that which will be taught before teaching and learning. Pupils then know what is required of them in terms of subject matter to be acquired. Learners need not out guess the teacher to realize what is expected as to precise objectives to be achieved.

According to Popham, the learning opportunities chosen by the teacher must contain only that which is in the stated objective(s), no more and no less. Evaluation of pupil attainment in mathematics needs to be done in terms of the measurably stated objectives. Thus a very close alignment indeed is in the offing among the objectives, the learning opportunities, and the evaluation procedures. Validity, a measurement term, is in evidence if the evaluation techniques harmonize with the stated objectives in mathematics.

James Popham with his stress placed upon behaviorism as a psychology of instruction in teaching mathematics advocates the following:

1. vague hazy objectives need to be eliminated or rewritten so that a sharp focus exists in terms of what will be taught.

2. learning opportunities must be very carefully chosen since each needs to guide pupils to attain that which is in the stated objective.

3. evaluation procedures should ascertain if each pupil has attained the precise objectives.

4. a different teaching strategy needs to be used if a learner did not achieve an objective.

5. sequence of objectives is arranged by the mathematics teacher.

Dr. Popham places extremely strong emphasis upon choosing precise, measurable stated objectives for instruction. He places little stress upon choosing learning opportunities, except that they should match up very precisely with the stated objectives. Evaluation is done strictly in terms of what is mentioned specifically in each objective for pupil attainment.

Robert Gagne' and Task Analysis of Objectives

Robert Gagne (1984) is a leading psychologist in education who recommends a behavioristic approach in teaching; however, his thinking is more open-ended as compared to Drs.Skinner and Popham. Gagne's eight sequential steps of hierarchical learning for pupils may be of considerable help to teachers in planning the mathematics curriculum. The eight steps of sequential learning for pupils are the following: signal learning, stimulus- response, chaining, verbal association learning, multiple discrimination, concept learning, rule learning, and problem solving as being the most complex form of achievement. Gage' was a former mathematics instructor and found task analysis as being a very appropriate way of determining sequence for pupils. I will comment on a few of the levels I believe to be especially relevant in teaching. Stimulus- response psychology is very relevant for mathematics teachers to consider. For example, once pupils attach meaning through the use of manipulative materials that 7+6 and 6+7= 13, this addition fact may be committed to memory. Thus on a flash card or computer program the stimulus is 7+6 or 6+7=.... If correct, pupils should respond with the answer being 13. Drill and practice should not be used prior to meaningful learning by pupils. But, once meaning is there, pupils may need to associate the stimulus and the response in a somewhat rote manner. Gagne's step of chaining might involve pupils using a series of concrete and semiconcrete materials to indicate and show that 7+6 and 6+7 =13. Which materials might these be? Sticks, corn and bean seeds, paper squares, and buttons, among other items, may be used by the learner in sequence to show a set of seven and a set of six and by joining the two sets together obtain a set of thirteen markers. The commutative property may also be shown by a learner. Chaining is involved in that the pupil used diverse materials to show the value of two addends. Multiple discrimination, in the Gagne' hierarchy of objectives stresses pupils noticing differences and likenesses in ongoing lessons and units of study. Analysis is involved here in that pupils separate the relevant from the irrelevant such as in seeking solutions to story or word problems. To do so indicated the need to make separations from what is needed to what is unnecessary. I believe the last three terms used by Gagne' are very significant in planning the mathematics curriculum. Thus concept learning is very relevant. It takes a variety of learning opportunities using different materials of instruction for pupils to understand concepts such as addition, subtraction, multiplication, division, inverse operation, radius, radius squared, radius cubed, and exponents. Understanding concepts are needed on the pupil's part in order that sequential achievement is possible in mathematics.

Gagne's principle or rule learning indicates that learners relate concepts so they become usable. A rule or principle such as "to find the area of a circle, square the radius and multiple by the value of pi" is necessary in a specific situation; otherwise pupils could not ascertain the area of a circle. The last idea in Gagne"s hierarchy is problem solving. Thus principles or rules are needed to understand how to solve the problem of determining the area of a square, triangle, or parallelogram.

A strong point in Gagne's hierarchy of objectives is that the teacher needs to go back a step or level if a pupil does not understand what is to be done. For example, if a pupils cannot solve a problem, perhaps he/she does not attach meaning to the involved rule or principle. If the rule or principle is a stumbling block to the pupils progress, he/ she may need to go back to learning the meaning of the inherent concepts within the rule or principle.

Jerome Bruner and the Structure of Knowledge in Mathematics

Jerome Bruner, professor from Harvard University, advocated a structure of knowledge approach in teaching mathematics. The structural ideas in mathematics would be identified by professional mathematicians in their academic area of specialty. These professional mathematicians then choose key or main ideas for pupil attainment. The structural ideas may be used again and again by learners as they proceed to more complex learnings on sequential grade levels. In mathematics then pupils may attain the following in increased levels of complexity:

1. commutative and associative properties of addition and multiplication.
2. distributive property of multiplication over addition.
3. property of closure.
4. subtraction as the inverse operation of addition.
5. division as the inverse operation of multiplication.

The above examples of structural ideas can be emphasized on sequential grade levels at increasing levels of complexity. For example, first grade pupils may learn that 4+3=7 and 3+4=7; this stresses the commutative property of addition. At a higher grade level, fifth grade pupils may learn meaningfully that 18,996 + 38,649= 38,469+ 18,996.

Jerome Bruner stressed the use of three kinds of materials in teaching mathematics to pupils. In sequence, these would be enactive, iconic, and symbolic. Enactive materials emphasize the use of concrete materials and other objects for learner manipulation in a hands on approach in learning. Second, pupils learn through the use of iconic materials which include pictures, illustrations, video tapes, video discs, slides, filmstrips, and other audio visual aids. Third, Bruner stresses the use of symbolic materials, such as printed content in textbooks, library books, and other abstract content. This sequence in pupil learning then emphasizes the teacher using concrete, semiconcrete, and abstract materials in teaching.

Jerome Bruner advocates the use of inductive methods of instruction in which pupils discover structural or major academic ideas of a discipline. To emphasize Bruner's approach in teaching, the teacher should attend to the following:

1. the teacher needs to have an excellent knowledge of the structure of knowledge since these key ideas become objectives for learner attainment.

2. to achieve objectives on the pupils' part, the teacher needs to sequence learning opportunities in that individuals experience the enactive, the iconic, and the symbolic in that order.

3. the teacher must appraise pupils to ascertain how many of structural knowledge objectives are being attained by pupils in a spiral curriculum. With a spiral curriculum, pupils meet up again again in increasing levels of complexity the structural ideas which serve as objectives of instruction.

4. the teacher needs to become a quality asker of questions involving the ongoing mathematics lesson so that pupils can truly learn in an inductive manner. Inductive teaching then assists pupils to achieve the structural ideas.

5. inductive teaching in mathematics must be used together with the enactive, iconic, and symbolic materials of instruction.

Jean Piaget and Developmental Psychology in Mathematics

Jean Piaget studied pupils in clinical settings for over forty years in Switzerland. He identified different stages that pupils go through in the maturation process. The first stage called the Sensorimotor Stage occurs from birth to two years in the infant's life. Here, parents need to have objects for the young child to manipulate and experience in a friendly environment. The child then experiences and perceives objects such as toys in the real environment. He/she may touch, smell, and see the objects. Listening to sounds made by these objects is also salient in sensorimotor learning.

The preoperational stage of development of the child roughly occurs from ages two to seven years. Here, the young child perceives one variable largely. Thus the preoperational child when viewing two tumblers of the same brand name and size as having an equal amount of water in each, if this is the case. Now, in front of the child, one of the two tumblers of water is poured into a taller thinner tumbler. The child is asked which has more water inside the tumbler. The preoperational pupil will answer the taller thinner tumbler does. The child perceives one variable in that one tumbler is taller than the other and therefore contains more water. If two spheres of clay are held in front of the preoperational child and both are identical in amount, the child will say neither has more clay in it than the other. But, if the experimenter flattens one sphere in front of the child, he/ she will say that the flattened clay has more in it than does the sphere of clay. Again, the preoperational child perceives one variable only and that being the flattened piece of clay is longer than the spherical lump of clay. Teachers of kindergarten and first grade pupils need to be aware that preoperational pupils lack maturation to notice that there is more than one variable to objects being observed. Preoperational pupils are perceptionally oriented. How something looks to the child is the correct perception or view. They tend to center on one variable such as the larger the area that one of two sets of marbles is placed in, even though both sets have an equal number of marbles, the larger the number of members of that set in the enlarged area. Thus if a set of six marbles is placed in a larger area, it will have more marbles than a set of six placed in a smaller region.

From ages seven through eleven, Piaget, in his research, found that these learners still needed concrete objects to learn from. Piaget called this the stage of concrete operations. Here, the learner has matured to emphasize reversibility. Thus the pupil may notice that the order of addends can be changed and yet the sum stays the same. Or, the concrete operations pupil learns that there are number families such as 7+5= 12 and 5+7= 12, which can be undone through subtraction within that number family such as 12-5=7 and 12-7=5. Reversibility also indicates that one can go back to an earlier stage of working on a project or activity and come back to the original starting point. One may go back (reversibility) to an earlier stage in unit teaching to further analyze what was done. One can also reverse to the original stage prior to emphasizing reversibility. Thus the concrete operations development pupils may perceive several variables when reversibility is in evidence.

Additive composition is also a part of the learner's stage of concrete operations. With additive composition, the pupil in perceiving numerous variables, may define, for example, what the identity elements are for addition and multiplication. There are numerous descriptions which can be given in the definition indicating again the pupil's ability to focus on several items at one time. All the definitions possible add up to a sum pertaining to the identity elements for addition and multiplication.

The principle of associativity is also a part of the concept the stage of concrete operations. With associativity, the pupil can add three or more numbers in any order. Or three or more factors may be multiplied in any order and the product is the same. Many tasks may also be taken up in any order and the results are the same or similar. In all facets of the pupil being in the stage of concrete operations, the teacher still needs to refer to and use concrete materials along with the abstract being emphasized.

At about twelve years of age, pupils enter the stage of formal operations. At the stage of formal operations, learners might be able to think abstractly in mathematics without reference to concrete materials of instruction. Learners in all stages of development need to operate or focus on what is being learned for learning to really take place.

There are numerous implications for teaching mathematics when using Piaget's research in teaching- learning situations. These include the following:

1. the teacher must study the maturational levels of pupils in order to know what and how to teach these learners.

2. there can be much wasting of time in teaching what the maturational level of the involved pupil is not ready for. Then too, the teacher must teach what the maturational level of the pupil is ready for in mathematics. Otherwise time slips by without the learner attaining as much as possible.

3. hastening the readiness of a pupil for learning mathematics does not work. The maturational level will indicate what can/cannot be taught.

4. there needs to be an adequate amount of concrete materials available for teaching since through the age of eleven, the stage of concrete operations is still in the offing.

5. securing attention for learning is salient since learners do not achieve unless they mentally operate upon the content being presented.

According to Piaget and Imhelder (1969), there are definite factors that that impinge upon pupils as they progress in intellectual development. These are biological maturation; interaction with experiences in the natural environment; social activities; and homeostasis, a balance between the self and experiences in the physical environment.

Biological maturation stresses pupils going through the stages of sensorimotor, preoperational, concrete operations, and formal thought. However, there are factors that influence these stages of biological maturation. One factor is pupils interacting with the natural environment. The richness of experiences here has much to do with learners developing biologically. Thus a stimulating environment in mathematics definitely affects progression in biological development. Working with others or being in groups that stress collegiality and its influence on both biological and the affects of the natural environment. Certainly, pupils learn much from each other pertaining to the world of mathematics. In supervising student teachers and cooperating teachers, I notice how pupils might affect each other very positively in ongoing lessons and units in mathematics. For example, in one class it was difficult for a pupil to understand and attach meaning to why the divisor is inverted and multiplication is stressed in the division of fractions. When this pupil and three others worked together in cooperative learning, one pupil made it very clear as to why the divisor is inverted and then multiplication occurs in the division of fractions.

Homeostasis emphasizes feelings of satisfaction that a solution has been found to a problem. Thus there is balance between the individual and his/ her environment. Equilibration has then occurred. In the previous example, when a learner understood what is involved when fractions are divided with the "invert the divisor and multiply" rule, the pupil also has reached a state of homeostasis at that point. Homeostasis may be followed again by a desire to know a new fact, concept, and/ or generalization. A good teacher will guide pupils to reach a state of disequilibrium so that an inward desire to learn is involved to seek new information and subject matter.

Piaget emphasizes that what is learned is grouped together in schemas. These schemes provide key ideas upon which future learning of the pupil is based. Schemas are also called structural ideas. Structural ideas or schemas are patterns of behavior of the individual. The pupil who has been actively involved in learning that 6+4= 10, may now use these learnings to achieve the new to be stressed such as 6+5= .... and 5+6=... New content might then fit into the older preexisting structures. In other words, previous content acquired in mathematics now sets the stage to learn more of higher decade addition. The involved process is called assimilation. There had to be accommodation so that the old and the new content might be blended.

Piaget (1971) continually emphasizes active involvement of the learner in the mathematics curriculum when writing the following:

> if we desire ... to form individuals capable of inventive thought and of helping the society of tomorrow to achieve progress, then it is clear that an education which is an active discovery of reality is superior to one that consists merely in providing the young with ready made wills

to will with and ready made truths to know with.

John Dewey and Problem Solving in Utilitarian Situations

John Dewey (1859- 1952) advocated a utilitarian mathematics curriculum in which school and society would be related. Thus what is useful in society should provide the basis for the school curriculum. Thus in mathematics, pupils with teacher assistance identify a problem area. The problem is significant to the learner. He/ she feels a definite need to find needed solutions. The problem then must be adequately delimited so that an answer can be found. Data or information is acquired in answer to the identified problem. The answer is tentative and subject to change due to further testing of the results. John Dewey did not consider textbook problems as being lifelike and reality based. Predetermined questions raised by the teacher and objectives written prior to instruction for pupils to attain do not stress problem solving. Rather within context in an ongoing unit of study preferably, the learner or a committee of pupils choose a problem in mathematics which is vital to solve. This is a practical problem to solve which emphasizes being useful and stresses application of content/skills acquired (Ediger, 1997).

Problem solving then emphasizes the useful and the utilitarian in the pupil's life in the school setting. Mathematics is a curriculum area that can truly emphasize that which is functional. Thus situations such as the following may stress a practical mathematics curriculum with problem solving involved:

1. measuring ingredients for a representative food dish of a foreign nation being studied in social studies.

2. planning and preparing a holiday meal in school whereby each pupil brings a certain amount of a food item.

3. averaging scores of the number of words spelled correctly by a pupil from six sequential weeks of spelling test scores.

4. developing a line graph from pupils individual birth dates in a calendar year.

5. operating a simulated supermarket in the classroom using real or toy money.

The above are merely suggestions for a reality based mathematics curriculum. One needs to remember that John Dewey advocated that problems come from pupils and not from an extrinsic source. The teacher guides learners in selecting and solving problems. John Dewey believed that pupils liked to work on committees rather than individually in solving problems. Learners, too, desired to find out on their own instead of being told how to locate an answer. Pupils were to be active, not passive recipients of knowledge. Creative behavior is preferred much more so as compared to conformity endeavors.

In summarizing John Dewey's problem solving approach in teaching pupils, the following are salient points:

1. activity centered approaches are emphasized in teaching in that pupils are the focal point of the curriculum.

2. mathematics stresses that pupils with teacher guidance select relevant lifelike problems which need solving.

3. a learning by doing, not passivity on the part of pupils, is a must for learning to accrue in mathematics.

4. purpose and interest on the learner's part make for learner effort and perseverance in solving problems.

5. the role of the teacher is to encourage, help, and assist pupils in attaining solutions to problems.

In Summary

There are numerous psychologists who may provide teachers with guidance in teaching mathematics. B.F. Skinner advocated a highly structured curriculum in which pupils would make few errors when achieving objectives arranged in an ascending order of difficulty. The programmer arranges the frames of learning in mathematics by using a sequence of read, view an illustration, respond, and check order. A psychology of behaviorism is emphasized here. Responses are either correct or incorrect as given by pupils individually.

James Popham believes in using measurement driven instruction (MDI) with the objectives stated behaviorally. The stated objectives leave no leeway for interpretation. The mathematics teacher then provides learning opportunities which contain only that which is in the stated objective. Appraisal is done in terms of the stated objective to determine if learners individually have been successful achievers.

Robert Gagne' emphasizes a hierarchical arrangement of teacher written objectives whereby the learner attains each in ascending order of complexity. Should an objective in mathematics be too difficult to achieve, the teacher needs to assist pupils individually to go back to a previous goal so that background information may have been attained. Then, the pupil should be ready sequentially to achieve the original objective. I will review the last three objectives Gagne' stressed in curriculum development. These are in sequence: concept development, attaining generalizations, and solving a problem, by the learner. The mathematics teacher writes the objectives for pupil achievement. Dr. Gagne', a former mathematics teacher, found that the teacher needed to go back to an earlier level of achievement if a pupil could not attain the preset objective being stressed in the curriculum. Thus if a learner did not understand how to solve a mathematics problem, he/she might need to go back to studying the related generalization that is inherent in the problem. Should the learner not understand the generalization, he/she may need to study the related concept(s). Once the concept(s) are understood, the pupil is ready to be taught the related generalization. If the generalization is meaningful, the involved pupil might then be ready to solve the problem. Robert Gagne' emphasized going back to an earlier level of achievement if the pupil did not attach meaning to what is presently being taught. Sequence in learning mathematics is very important to Robert Gagne'. All good teachers of mathematics prize highly if content, skills, and attitudes are learned sequentially by learners. This may mean reversing to an earlier level of attainment if a pupil does not understand or comprehend that which is being taught presently. The sequence may also pertain to a learner being taught more complex subject matter in mathematics if presently the objectives have been achieved.

Robert Gagne' advocated a hierarchy of objectives for pupils to attain in mathematics which meet the following standards:

1. the objectives are arranged so that each pupil may attain an appropriately ordered set of goals which move from the easier to those increasingly more complex.

2. the teacher may place another objective between two others if a pupil makes an error at that point.

3. the problems to be solved tend to be more abstract than those advocated by John Dewey.

4. the objectives are determined prior to instruction.

5. quality sequence in mathematics makes for fewer learner errors when being engaged in a lesson or unit of study. The teacher sequences or orders the objectives for learner attainment.

Jerome Bruner emphasized that pupils on any grade level attain structural ideas in mathematics. Mathematicians at the university level select and agree upon these ideas. The structural ideas are available to teachers who teach pupils. Inductively, pupils are to achieve these structural ideas on an increasingly difficult level as they progress through sequential levels of attainment.

Jean Piaget stressed the importance of pupils going through specific maturational levels such as the sensorimotor, preoperational, concrete, and abstract levels. Biological maturation is salient when teachers ascertain what should be taught to pupils. The stage of concrete operations, for example, has associativity as one of its subcategories. The associative properties of addition and multiplication, using concrete materials, should be taught at this stage (ages seven to eleven) of learner development.

John Dewey advocated a problem solving approach in which pupils identify and solve problems. The problems are lifelike and practical. The useful and the utilitarian are emphasized. Dewey placed strong emphasis upon democracy as a way of life. Democracy then is more then a political system or a way of governing individuals. Democracy is a way of associating with others and a means of communication. It is a means of involving all who will be affected by a given decision (Dewey, 1916). Democratic means are needed to identify and solve problems. These problems need to be reality based and practical in the societal arenas. Pupils then need guidance to select and solve life-like problems in mathematics. To stress democracy in the school and classroom settings, learners should work cooperatively in problem solving endeavors in mathematics.

The teacher of mathematics needs to use a psychology of teaching which will guide the learner to achieve as optimally as possible. There are diverse psychologies available to provide guidance in helping pupils achieve, grow, and learn in mathematics. Pupils need to be able to use what has been acquired. Meaning is then attached to facts, concepts, and generalizations achieved in mathematics. The National Council Teachers of Mathematics in 1989 developed excellent criteria, goals, and objectives for pupils to achieve. These are listed in their book Curriculum and Evaluation Standards for School Mathematics. The psychologies discussed above may well be used to guide pupils in goal attainment from those listed in Curriculum and Evaluation Standards for School Mathematics.

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